Microwave Modulator Requiring Minimum Modulation Power*

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Summary—A microwave modulator was devised which requires a minimum of modulation power and which has very high modulation efficiency. The modulator functions as a switch, which opens and closes a transmission line proportional to a time function which represents information in digital form. The modulator is a varactor diode operating in the reverse bias region. To obtain the required modulation characteristics, the diode was switched between two resonant modes. It was operated alternatively in series and in shunt resonance by varying the diode's capacitance. To obtain the resonant modes, tuning elements were added in series and in shunt with the varactor diode.

At the design frequency of 2.2 Gc the semiconductor modulator has an insertion loss of less than 1 db in the transmission mode and more than 24 db in the rejection mode. In the transmission mode Q_L is close to one; in the rejection mode Q_L ranges from 30 to 50.

The consumption of modulation power is extremely low. For a modulation voltage of square waveform with the fundamental frequency of 1 Mc, the dissipated modulation power is below 10^{-8} watts. The modulation efficiency is nearly one, since the response time of the varactor diode which is operated in the space charge region is below 10^{-11} sec.

Introduction

A SEMICONDUCTOR modulator has been devised for a modulated microwave space antenna array. In the space antenna, which is based on the Van Atta principle, the modulators are placed in the transmission lines that connect conjugate array elements. The modulator functions as a switch; it opens and closes the RF transmission line proportional to a time function that represents information in digital form. In a high gain array, a large number of modulators are operated in shunt. It is essential, therefore, that the modulators be small and light and that they require a minimum amount of modulation power. In addition, high reproducibility of their characteristics is necessary.

The modulator is a varactor diode placed in shunt in an RF transmission line. To meet the minimum modulation power requirement, the diode is operated in the reverse bias region. Therefore, it differs from semiconductor modulators that have been earlier reported,[1–3] since these are driven from the negative bias into the positive bias region.

To function as a modulator the RF admittance of the varactor diode must be varied from a very low to a very high value. In the reverse bias region, the range of admittance variation is limited. In order to yield the required modulation characteristic, the diode has to be switched between two resonant modes. To do so, tuning elements are added in series and in shunt with the diodes. The RF admittance of the modulator circuit, which is in shunt with the transmission line, will be operated alternatively in series and shunt resonance by varying the diode's capacitance. This resonant mode of operation yields the required modulation characteristic but necessarily narrows the modulator bandwidth. It has, however, the added advantage that variation of the characteristics between individual diodes can be compensated for by adjusting the tuning elements.

A description of the modulator, and an analysis and experimental results on its RF performance, will be given, followed by an analysis of its modulation power requirement.

Modulator Analysis

The performance of the modulator circuit can be described best from its equivalent circuit shown in Fig. 1. The circuit elements of the *p-n* junction, when operated in the reverse bias region, are the capacitance C_D of the space-charge transition region, which is shunted by the resistance of the transition region R_p. Both are in series with a small resistance R_s. The transition region widens as the reverse bias is increased; consequently, its capacitance decreases. The shunt resistance R_p is extremely large for silicon diodes and will be

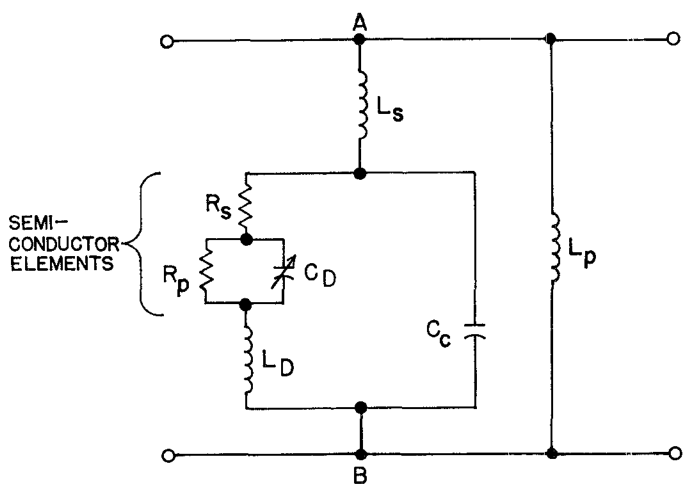

Fig. 1—Modulator equivalent circuit.

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[1] R. V. Garver, "Theory of TEM diode switching," IRE Trans. on Microwave Theory and Techniques, vol. MTT-9, pp. 224–238; May, 1961.

[2] M. Bloom, "Microwave switching with computer diodes," *Electronics*, vol. 33, pp. 85–87; January 15, 1960.

[3] R. L. Michael, "Design of a high speed diode switch," *Electronic Design*, vol. 9, pp. 106–111; August 30, 1961.

neglected in the analysis. In addition to the circuit elements of the semiconductor, the equivalent circuit contains the cartridge capacitance C_c, which is small compared to the transition capacitance and the whisker inductance, L_D. A series inductance L_s and a shunt inductance L_p are added in the modulator circuit.

With no bias applied to the diode, the branch A-B is in series resonance at the design frequency. Its impedance is given by

$$R_{A-B} = \frac{R_s}{\left[1 - \omega C_c\left(\omega L_D - \frac{1}{\omega C_D}\right)\right]^2 + (\omega C_c R_s)^2}. \quad (1)$$

R_{A-B} for the varactor diode will always be somewhat smaller than R_s.

When reverse bias is applied to the diode, the diode capacitance is reduced to approximately half its value, and the series resonant circuit will become detuned. The imaginary component of the branch A–B, which is given by

$$X_{A-B} = \omega L_s + \frac{\left(\omega L_D - \frac{1}{\omega C_D}\right) - \omega C_c\left(\omega L_D - \frac{1}{\omega C_D}\right)^2 - \omega C_c R_s^2}{\left[1 - \omega C_c\left(\omega L_D - \frac{1}{\omega C_D}\right)\right]^2 \omega + (C_c R_s)^2}, \quad (2)$$

becomes capacitive. The imaginary component of branch A–B forms a shunt resonant circuit with the inductance L_p. This circuit, which resonates at the design frequency represents a high impedance. The series resonance of branch A–B is independent of the shunt inductance, L_p. Consequently, the tuning of the series and the shunt resonant circuit can be accomplished in two successive steps.

In the antenna, the modulator is operated as a bilateral device; its modulation characteristics can be defined best by its scattering matrix. Because the diode is a lossy circuit element, the scattering matrix is not unitary; it is given by

$$\begin{vmatrix} S_{11} & S_{12} \\ S_{12} & S_{11} \end{vmatrix} = \begin{vmatrix} \Gamma & \frac{2}{2+Y} \\ \frac{2}{2+Y} & \Gamma \end{vmatrix} \quad (3)$$

where

$$\Gamma = -\frac{G^2 + 2G + B^2 + j2B}{(2+G)^2 + B^2}$$

$$G = \frac{1}{Y_0} \frac{R_{A-B}}{R_{A-B}^2 + X_{A-B}^2} \quad (4)$$

$$B = B_c - B_L = \frac{1}{Y_0}\left(\frac{-X_{A-B}}{R_{A-B}^2 + X_{A-B}^2} - \frac{1}{\omega L_p}\right)$$

$$Y = G + jB$$

Y_0 = characteristic admittance of main transmission line.

For zero bias at the design frequency,

$$X_{A-B} = 0$$
$$G \gg 1$$
$$B \ll 1.$$

The voltage reflection coefficient S_{11} becomes close to -1, and the voltage transmission coefficient S_{12} is close to zero. The incoming waves are reflected at the modulator. At negative bias,

$$B = 0$$
$$R_{A-B} \ll X_{A-B}$$
$$G \ll 1.$$

The voltage reflection coefficient S_{11} becomes very small, and the voltage transmission coefficient S_{12} approaches 1. The modulator represents a small admittance to the main transmission line; the incoming waves will be transmitted through the transmission line.

The bandwidth of the modulator in shunt with the transmission line, which is inversely proportional to the Q_{loaded} of the circuit, is given by

$$\frac{\Delta f}{f_0} = \frac{1}{Q_L} = \frac{G+2}{B_c} \quad (5)$$

where B_c represents the capacitive susceptance of the modulator circuit in (4).

In the modulated antenna, the phase, as well as the amplitude, of the transmission coefficient of the modulator is of importance, especially in the transmission mode. In the bilateral system, the resultant wave at each terminal pair of the modulator is the vector sum of the wave, which is passing by the modulator, and of the fraction of the wave incident from the opposite side, which is reflected at the modulator. The phase error is given by

$$\tan \Delta\epsilon = \frac{\text{Im}(S_{12}) + \text{Re}(S_{11}) \sin \Delta\phi + \text{Im}(S_{11}) \cos \Delta\phi}{\text{Re}(S_{12}) + \text{Re}(S_{11}) \cos \Delta\phi - \text{Im}(S_{11}) \sin \Delta\phi}. \quad (6)$$

It is determined by the imaginary component of the voltage transmission coefficient and depends on the phase of the voltage reflection coefficient and on the relative phase, $\Delta\phi$, which the waves traveling in opposite directions have at the modulator. In the modulator circuit, the possible phase error for the transmission mode is greatly reduced by having the Q_{loaded} of the shunt resonant circuit comparatively low (as outlined later), because in a low Q circuit, a change in operating frequency will result in only a small increase of S_{11} and Im (S_{12}).

The experimental model of the modulator circuit is shown in Fig. 2; it demonstrates the simplicity and compactness of the design. A pigtail-type varactor diode is connected between the center conductor and the side wall of a 50-Ω strip transmission line. The pigtail wire represents the series inductance of the

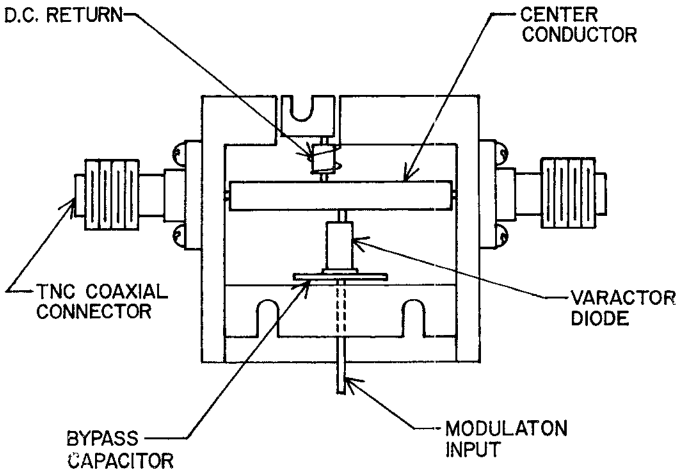

Fig. 2—Experimental model of modulator.

modulator circuit; its length is tuned for series resonance at zero bias. The shunt inductance, which is tuned for shunt resonance at the negative bias voltage, functions at the same time as dc and ac return.

The input and output of the modulator circuit are coaxial connectors. The transition from coaxial to strip transmission line is a step in the inner conductor of the transmission line corresponding with the change of geometry of the outer conductor. The varactor diode is: Microwave Associates, MA 4321A or MA 4321AA. Low temperature solder (Indium Corp. No. 1) was used to connect the diode between the center conductor of the transmission line and the bypass capacitor to avoid overheating of the diode.

The experimental model weighs 2.2 ounces and occupies 1.2 cubic inches. The size and weight can be considerably reduced in a unit for an aerospace mission because the varactor weight is only 0.0053 ounce.

In Fig. 3, the measured insertion loss characteristics of the modulator circuit, which was described before with the diode MA 4321A ($C_D = 0.88$ pf) are given by

$$L = 20 \log \frac{1}{|S_{12}|} \cdot$$

In Fig. 4 the measured insertion loss characteristics for the diode MA 4321AA ($C_D = 0.55$ pf) are shown. Both modulator circuits were tuned to the same frequency. There was comparatively good correlation between measured characteristics and values calculated from (1)–(4).

The bandwidth of the modulator circuit can be derived from (1), (2), (4), and (5). With no bias applied to the modulator $X_{A-B} = 0$; in addition, the denominator of (1) and (2) becomes close to one. Eq. (5) can be approximated by

$$\frac{\Delta f}{f_0} \approx \frac{R_s(1 + 2Y_0R_s)}{(X_{A-B})_c}$$

where $(X_{A-B})_c$ are the capactive reactances (negative terms) in (2).

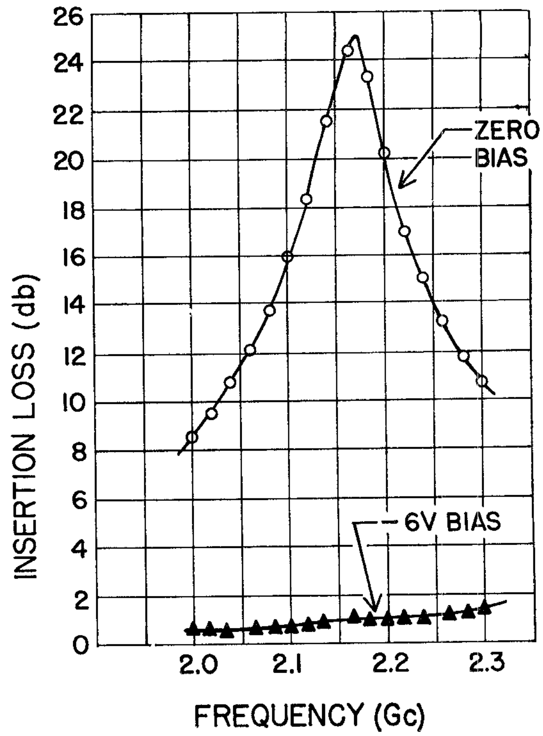

Fig. 3—Insertion loss characteristics of experimental modulator (0.88-pf varactor). MA 4322A Varactor: $C_D = 0.88$ pf (zero bias); $C_D = 0.37$ pf (-6 volts bias); $f_{co} = 79$ Gc (-6 volts bias).

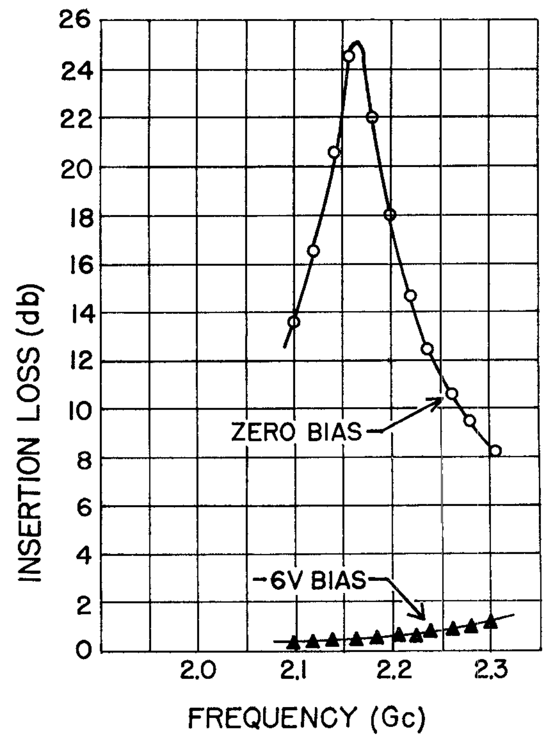

Fig. 4—Insertion loss characteristics of experimental modulator (0.55-pf varactor). MA 4321AA Varactor: $C_D = 0.55$ pf (zero bias); $C_D = 0.23$ pf (-6 volts bias); $f_{co} = 43$ Gc (-6 volts bias).

The bandwidth is determined primarily by the ratio of the series resistance of the diode to its capacitative reactance. The bandwidth is small, since the series resistance and the capacitance of the diode are very small. For the 0.88-pf varactor of Fig. 3, $\Delta f/f_0=0.035$, $(Q_L=28)$ and for the 0.55-pf varactor of Fig. 4, $\Delta f/f_0=0.019$, $(Q_L=52)$. The measured bandwidths in Figs. 3 and 4 agree reasonably well with the computed values.

At negative bias $R_{A-B} \ll X_{A-B}$ and (5) can be simplified to

$$\frac{\Delta f}{f_0} \approx \frac{R_{A-B} + 2Y_0(X_{A-B})^2}{(X_{A-B})_c}.$$

For the varactor diodes of Figs. 3 and 4,

$$\frac{2Y_0\,(X_{A-B})^2}{(X_{A-B})_c} \approx 1$$

and $\Delta f/\Delta f$ and Q_L are close to 1; the insertion loss characteristics at shunt resonance (−6 volts) is rather frequency insensitive, as can be seen from Figs. 3 and 4.

The phase error $\Delta\epsilon$ in the transmission mode of the modulator circuit was computed for the varactor of Fig. 3 from (6). For a change in the junction capacitance C_D of 10 per cent, the maximum phase error will not exceed 10°.

Modulation Efficiency and Modulation Power Consumption

The modulation voltage, which is applied to the ac terminals of the varactor diode, switches the operating point on the diode capacitance-voltage characteristic alternatively from zero to a negative bias voltage. In effect, then, the function of the driver circuit is to charge and discharge the space-charge capacitance of the diode. The efficiency of the driver circuit will be determined by the efficiency of the energy transfer from the modulation voltage generator to the capacitance of the diode. The energy transfer efficiency is given by[4]

$$\eta = \frac{\text{energy delivered to the varactor capacitance}}{\text{energy supplied by the source}}.$$

The equivalent circuit of the diode at modulation frequency (approximately 1 Mc) can be simplified. The inductive reactances can be neglected; in the ac equivalent circuit, the diode's resistance, R_s, will be in series with the capacitance of the space-charge region which is shunted by the cartridge capacitance. For the purposes of this analysis, the varactor capacitance will be assumed to be voltage independent and the shunt resistance, R_p, is neglected.

In charging and discharging the capacitance, power is dissipated in the series resistance of the varactor diode. The waveform for digital off-on modulation is nonperiodic; the highest power loss for digital modulation occurs when the off-on switching is a periodic function. Therefore, the power loss in the varactor diode will be derived assuming a periodic waveform which approximates a square wave.

In the frequency domain, the modulation voltage can be represented by the frequency of the on-off switching and its odd harmonics. The energy per unit time delivered to the capacitance and the power dissipated in the series resistance are the sums of energy per unit time and power loss for the frequency components contained in the square-wave modulation voltage.

In the modulator circuit, the series resistance is much smaller than the capacitive reactance; it can, therefore, be assumed that the current, i_c, is determined primarily by the capacitance of the diode. Because the amplitude of the voltage components in the square-wave spectrum decreases with harmonic number while the capacitive susceptance increases, the charging current will be the same for all frequency components in the square-wave spectrum.

The transfer efficiency, η, can be defined simply by the sum of reactances and resistances.

$$\eta = \frac{\sum_{n=1}^{n} \frac{1}{2n-1} \frac{i_c^2}{\omega_M(C_D+C_c)}}{\sum_{n=1}^{n} i_c^2 R_s + \sum_{n=1}^{n} \frac{1}{2n-1} \frac{i_c^2}{\omega_M(C_D+C_c)}}$$

$$\eta = \frac{1}{1 + \frac{2}{n\omega_M R_s (C_D + C_c) \sum_{n=1}^{n} \frac{1}{2n-1}}}. \tag{7}$$

In comparing (7) with the transfer efficiency given by Mostov,[5] it can be seen that the term

$$\frac{2}{n\omega_M} \sum_{n=1}^{n} \frac{1}{2n-1}$$

is equivalent to the rise and decay time, T, of the periodic modulation voltage, where this voltage contains n frequency components, the highest of which is $(2n-1)f_M$.

The efficiency of the energy transfer will be very close to 1 when the response time of the varactor diode is small in comparison to the rise and decay time of the modulation voltage. Because the response time of the varactor diode is extremely small, this condition will exist for all practical modulation voltages.

The response time of the varactor diode, when being operated in the space-charge region, is determined by the time constant of the RC circuit.

$$\tau = R_s(C_D + C_c). \tag{8}$$

[4] P. M. Mostov, J. L. Neuringer and D. S. Rigney, "Optimum capacitor charging efficiency for space systems," Proc. IRE, vol. 49, pp. 941–948; May, 1961.

[5] Mostov, *et al.*, *op. cit.*, p. 944, Eq. (9).

The time constant of the varactor diode is approximately $\tau \approx 3 \times 10^{-12}$ sec.

Assuming that the fundamental frequency of the modulation voltage is 10^6 sec^{-1}, then the rise and decay time of a conventional square-wave generator will not be below 0.5×10^{-7} sec. This means the spectrum of the modulation voltage will contain approximately 10 frequency components. The efficiency of energy transfer for these values becomes

$$\eta = \frac{1}{1 + 1.2 \times 10^{-4}}.$$

Obviously, the energy transfer efficiency will, in all practical cases, be very close to 1.

In the modulated antenna, the transfer efficiency will remain the same, independent of the number of varactor diodes operated in shunt because in the equivalent circuit of N diodes operated in shunt the series resistance is R_s/N, while the capacitance is $N(C_D+C_c)$, and the over-all time constant remains $\tau = R_s(C_D+C_c)$.

The rise and decay time of the modulation voltage will not be determined by efficiency considerations. However, the rise and decay time does become a limiting factor in keeping the power dissipation in the series resistance to a minimum. The power dissipation is given by

$$\sum_{n=1}^{n} i_c^2 R_s = n i_c^2 R_s. \tag{9}$$

The power loss increases proportionately to the number of frequency components contained in the spectrum of the modulation voltage. Assuming that the operating points on the diode characteristic are zero and -6 volts and $R_s = 3\ \Omega$, then the power dissipated in the modulation circuit by switching the diode at a rate 10^6 sec^{-1} becomes

$$P = n \cdot 10^{-9} \text{ watts.} \tag{10}$$

(For $n=10$, $P=10^{-8}$ watts.)

When N diodes are operated in shunt, then the power loss increases in proportion to the number of diodes.

$$P = N \cdot n\, 10^{-9} \text{ watts.} \tag{11}$$

In the derivation of the power loss, it was assumed that the space-charge capacitance of the diode remains constant. However, because the capacitance decreases for increasing negative bias, the dissipated power will be smaller than that given in (10) and (11).

In the computation of modulation efficiency and the power dissipation, the shunt resistance R_p in Fig. 1 was neglected. This can be done when $R_p > 10^9\ \Omega$ since then the power dissipated in the series resistance, which is given in (10) for a switching rate of 10^6 sec^{-1} and $n=10$, is larger than the power dissipated in the shunt resistance R_p. Most of the measured values of R_p for the varactor diodes MA 4321A and MA 4321AA were higher than $10^9\ \Omega$. For $R_p = 10^9 \Omega$ or less, the effect of R_p has to be considered in the analysis, since for $R_p = 10^9 \Omega$ the dissipated power in R_p becomes equal to the power dissipated in the series resistance for a switching rate of 10^6 sec^{-1} and $n=10$.

When the shunt resistance becomes effective, the response time of the varactor diode increases and (8) then becomes

$$\tau = \left(R_s + \frac{1}{R_p[\omega(C_D + C_c)]^2}\right)(C_D + C_c).$$

It is interesting to notice that the term which was added to (8) is inversely proportional to the square of the modulation frequency. This means that the modulation efficiency in (7) will hardly be affected by the shunt resistance. For low modulation frequencies, τ will increase but the product $\omega_M \tau$ will be very small; for higher modulation frequencies the response time τ decreases and the product $\omega_M \tau$ will remain very small.

The power loss of the modulator circuit could not be measured directly, since in the experimental model of the modulator the capacitance of the bypass capacitor was many times the diode capacitance. The only parameter which was actually measured was the current as a function of time which is required to drive twelve of the experimental modulators including their bypass capacitances when operated in shunt in a modulated array. From the measured current, the power loss in the modulator ensemble can be derived. The power loss in the modulator circuits themselves will then be deduced from the power loss in the modulator ensemble. Since the instantaneous current as a function of time was measured, the power loss in the modulator ensemble must be derived using the transient analysis rather than the periodic analysis which was given above.

The characteristics of the modulator ensemble and its driver voltage are summarized in the following. The source resistance of the driver circuit is 65 Ω, the effective series resistance of the twelve modulators when operated in shunt is 0.25 Ω, the effective capacitance of modulators and bypass capacitances is 500 pf. The driver voltage is a square wave which changes the bias to the diode from zero to -6 volts, the rise time from zero to -6 volts is 0.1 μsec, the width of the voltage pulse is 1 μsec, and the decay time is 0.1 μsec.

The current of the modulator ensemble which was measured increases its value continuously during the rise time of the voltage pulse and reaches its peak value of 30 ma when the negative voltage reaches its peak value of -6 volts.

The power loss in the modulator ensemble is the energy dissipated in the effective series resistance of the 12 shunted diodes per period; it is given by

$$\frac{W}{T} = 2\,\frac{\left(\frac{AC}{T_a}\right)^2 R(T_a - \tau)}{T} \tag{12}$$

where

A = the peak amplitude of the driving voltage (6 volts)
C = the effective capacitance of the 12 shunted diodes and their bypass capacitances (500 pf)
T_a = the rise and decay time of the driving voltage (0.1 μsec)
R = the effective resistance of the 12 shunted diodes (0.25 Ω)
R_G = source resistance (65 Ω)
$\tau = (R+R_G)C \approx 0.3\ T_a$
T = period of the square wave (2×10^{-6} sec).

Since the peak current for $t=T_a$ is given by

$$i_{\max} = \frac{AC}{T_a},$$

(12) becomes

$$\frac{W}{T} = 2\frac{i_{\max}^2}{T} R(T_a - \tau). \tag{13}$$

The power loss of the modulator ensemble computed from (13) is 1.5×10^{-5} watts. The power loss in the modulator circuits themselves can now be derived from (12) when C is made the effective capacitance of the 12 shunted diodes alone. When C is assumed to be 12 pf, the power loss in the 12 modulator circuits alone, derived from (12), becomes 10^{-8} watts. (In a later model of the semiconductor modulator, a complete redesign of the bypass capacitance will bring the capacitance of the modulator ensemble close to the capacitance assumed for the modulator circuits alone.)

The power loss of 12 modulators operated in shunt when calculated from (11), is 2.4×10^{-8} watts, assuming that the fundamental frequency in the modulating square wave is 0.5×10^{6} sec^{-1} and the highest harmonic is 0.8×10^{7} sec^{-1} ($n=8$). [The fundamental frequency is half the value which was assumed to derive (10) and (11).] There is very good correlation between the power loss which was derived from the measured current and the power loss which was calculated from (11).

In conclusion, by utilizing the resonant operation of a reversed biased varactor diode in a shunt configuration, the experimental modulator achieves the performance objectives of good modulation performance with extremely low modulation power consumption. At the design frequency, the semiconductor modulator has an insertion loss of less than 1 db in the transmission mode and more than 24 db in the rejection mode. The insertion loss in the transmission mode is comparatively insensitive to changes in the operating frequency; the bandwidth in the rejection mode ranges from 2 to 4 per cent.

The consumption of modulation power is extremely low; the modulation power for 100 diodes when operated in shunt will not exceed 1 μw. The modulator has the desired simplicity, compactness, and light weight.

Acknowledgment

The assistance of E. Hurd in making the experimental measurements is gratefully acknowledged.

Correction

W. H. Eggimann, author of "Higher-Order Evaluation of Electromagnetic Diffraction by Circular Disks," which appeared on pages 408–418 of the September, 1961, issue, of these Transactions, has brought the following misprints to the attention of the *Editor*. The following expressions should read

$$a_2 = \frac{\mu_0}{30}\left\{30H_z + a^2\left(-13\frac{\partial^2 H_y}{\partial y\partial z} + 11\frac{\partial^2 H_x}{\partial x\partial z} - 8\frac{\partial^2 H_z}{\partial x^2}\right) - 9(ka)^2 H_z\right\} + j\frac{4a^2}{9\pi}\mu_0(ka)^3 H_z \tag{33}$$

$$b_2 = \frac{\mu_0}{30}\left\{-30H_z + a^2\left(13\frac{\partial^2 H_x}{\partial x\partial z} - 11\frac{\partial^2 H_y}{\partial y\partial z} + 8\frac{\partial^2 H_z}{\partial y^2}\right) + 9(ka)^2 H_z\right\} - j\frac{4a^2}{9\pi}\mu_0(ka)^3 H_z \tag{34}$$

$$\boldsymbol{P} = \frac{16}{3}\epsilon_0 a^3\left\{\boldsymbol{E}_{\tan} + \frac{(ka)^2}{30}\left(13\boldsymbol{E}_{\tan} - \frac{3}{k^2}\frac{\partial^2 \boldsymbol{E}_{\tan}}{\partial z^2}\right) - j\frac{8}{9\pi}(ka)^3\boldsymbol{E}_{\tan} + j\frac{1}{15\omega\epsilon_0}(ka)^3\nabla\times\boldsymbol{H}_z\right\} \tag{38}$$

$$I(n, m, \mu; \rho, \phi) = \sum_v A_v(n, m, \mu)\ \overset{2m}{\underset{2v}{P}}\left\{(a^2-\rho^2)^{1/2}\right\}\cos(2m\phi). \tag{56}$$

Eq. (38) here is equivalent to (38) in the paper.